



## Numerical solution of the problem on the impact plane non-stationary elastic waves by a cylindrical body

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### General Note



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## ABSTRACT

In this paper the impact of the non-stationary waves on the cylindrical body with a circular or rectangular cross-sections is discussed. The problem is solved in the flat setting. The numerical method when the impact load in the form of a unit function of Hevisayd. Obtained Numerical results.

**Keywords:** Unsteady wave Heaviside function, numerical method, a triangular element, the viscous boundary.

## 1. INTRODUCTION

For modern engineering practice in the construction of underground structures is very significant and important role played by research and analysis of wave phenomena occurring in environments with a variety of irregularities. The obtained results in this area are crucial to create the methods of calculation of dynamic effects on the structure and facilities, interacting with different types of soils. However, solving the problem can not be achieved without significant progress deeper their theoretical analysis. The main provisions of the dynamic theory of seismic stability (TPA) have been developed in the works [1,2,3,4], and others. These provisions are as follows [5,6,13]. We consider an arbitrary scheme underground network consisting of elastic rods (pipes, tunnels, trunks) and mating with high rigidity structures (manholes, subway stations and so forth.). The movement of the soil surrounding the pipe, with the earthquakes is represented as a traveling wave of variable intensity. In this formulation of the problem is considered only process associated with fluctuations in the pipeline in the ground without taking into account the volume of the oscillating ground mass [2]. This takes into account the soil resistance, the friction sliding rods in the ground. In this formulation, the problem is solved by a set of differential equations describing the oscillations of the rods, taking into account the dynamic and kinematic coupling rods conditions. On the basis of the above-described computational model to study the effect of seismic waves on the lines experiencing longitudinal oscillations [12,14]. Among the most common method of calculation used in the calculation of underground pipelines, tunnel construction, using the finite element method (FEM) and the grid method. By the variational-difference methods include: the method of Bubnov-Galerkin, Ritz method and finite element method [11,16].

Let us dwell on the latter, which is found in the currently widely used to solve practical engineering problems. In the calculations made uneven breakdown of the computational domain into rectangular and triangular finite elements. This split deepened as it approaches the ground area adjacent to the pipe. Currently, there are well-designed software systems for analyzing plane and three-dimensional problems of linear and non-linear elasticity FEM [7,8,9,10,15,17,18,19]. Such problems can be solved on a multiply connected domain any outline (with the exception of the definition of stress-strain state in a small neighborhood of a singular point).

## 2. FORMULATION OF THE PROBLEM

The Cartesian coordinate system is considered flat area, which is set free round (or square) opening. We consider the reinforcement of the middle loop diameter to thickness ratio equal to ten. Prior to the time  $t = 0$  of rotation of the considered point of the mechanical system are at rest:

$$\begin{aligned} u|_{t \leq 0} &= 0; & \vartheta|_{t \leq 0} &= 0 \\ \frac{\partial u}{\partial t}\bigg|_{t \leq 0} &= 0; & \frac{\partial \vartheta}{\partial t}\bigg|_{t \leq 0} &= 0; \end{aligned} \quad (1)$$

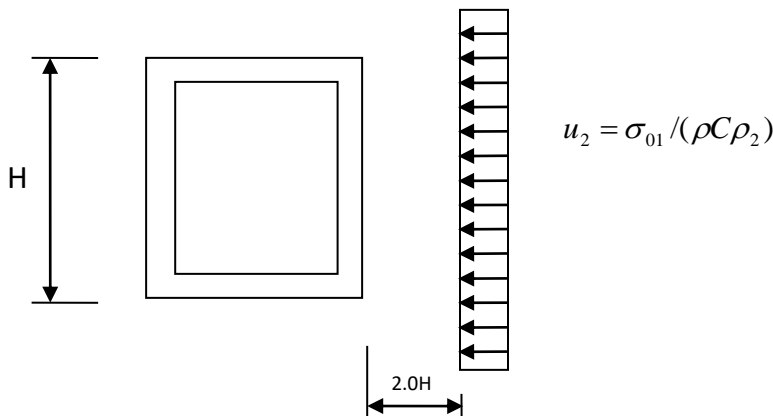
Starting from the time  $t \geq 0$ ,  $\Omega$  to the area in a limited volume of applied external load

$$u = \sigma / (\rho C) , \quad (2)$$

Where  $\sigma$  - the amplitude of the external loads;  $\rho$  - material density;  $C$  – rate review of longitudinal waves. For the time-dependent problems in a study of the conditions required to fulfill the principle of causality in the environment should be no movement outside the region bounded by the leading edge of the waves coming from sources of vibrations. The boundary conditions at the boundary of the computational domain for separating dynamic (seismic) influences. In solving problems for infinite elements of the infinite half-released study of the computational domain of finite size. The study area is sampled moreover there is a need of setting conditions on the border that would not show in the results of the solution due to reflections that occur long pi dynamic effects. Some researchers consider the solutions offered only at some distance from the boundary of [4,8], considering that the reflected waves can not reach this area during the reporting period of time. Sometimes it is advisable to enter into the calculated additional area of artificial damping increasing as we approach the border [7]. The paper Lismera [7] been proposed boundary conditions for finite computational domain, allowing to simulate an infinite medium. These boundary conditions are passed through the boundary wave computational domain without reflection, ie get the so-called standard viscous boundary. Setting the standard viscous boundaries it is carried out by replacing the reaction does not take account of the distributed loads of half-planes and  $\tau$ , calculation formula:

$$\sigma = \alpha \rho C_p u; \quad \tau = \beta \rho C_s v; \quad (3)$$

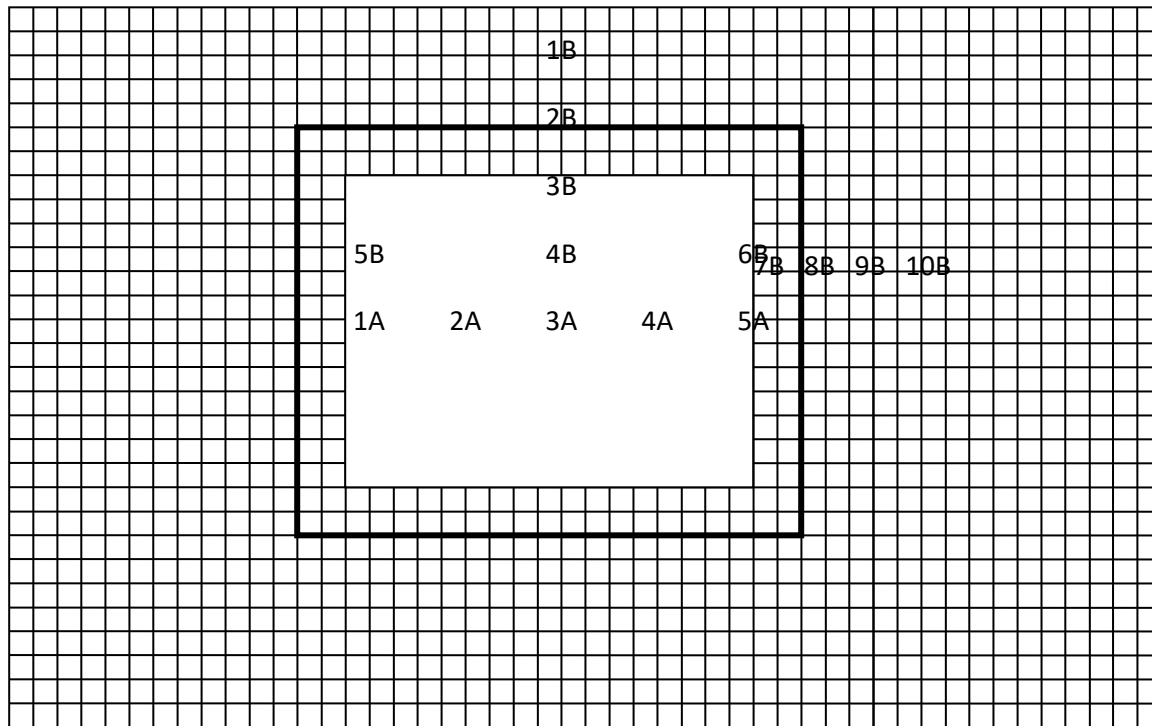
where  $u$  and  $v$  - velocity of points on the boundary of the body, respectively, the coordinates  $X_1$  and  $X_2$ .  $\alpha$  and  $\beta$  - limitless options;  $\rho$  - material density;  $C_p$  and  $C_s$  - velocity respectively of the longitudinal and transverse waves. Such conditions can be considered as the installation of the viscous dampers at the interface.



$\sigma$ . Effects of elastic wave to reinforce the square hole

$\rho$ . The impact of elastic waves on supported by a round hole

**Figure 1** Payment schemes



**Figure 2** The points at which the elastic stress given time

### 3. METHODS OF SOLUTION

FEM procedures provide for the transition from differential dependencies for the individual finite element to the global system of equations for the entire array. For linear problems of unsteady interaction of this system has the form [20]:

$$[M]\{\ddot{q}\} + [S]\{\dot{q}\} + [K]\{q\} = \{F\} - [p]\{\delta\} \quad (4)$$

here  $[M]$ ,  $[S]$ ,  $[K]$  – respectively mass matrix, damping system stiffness;  $\{\ddot{q}\}$ ,  $\{\dot{q}\}$ ,  $\{q\}$  – Entrenched vectors, velocity and displacement;  $\{F\}$  – vector of external loads;  $[p]$  – Matrix external dempfirovaniya. Matrichnoe differential equation (4) in the finite - difference form using Newmark method [21]

$$\begin{aligned} & \left(\frac{1}{\Delta t}\right)^2 [M](q^{j+2} - 2q^{j+1} + q^j) + \left(\frac{1}{2\Delta t}\right)[S](q^{j+2} - q^j) + [k][\beta q^{j+2} + (1-2\beta)q^{j+1} + \beta q^j] = \\ & = \beta F^{j+2} + (1-2\beta)F^{j+1} + \beta F^j q^j \end{aligned} \quad (5)$$

where  $j$ ,  $j+1$ ,  $j+2$  – past, present and future values of the variables;  $\beta$ -option is chosen from the condition of numerical stability and accuracy. In this example, it is accepted  $\beta=1/3$ . Thus, a system of linear algebraic equations, which is solved by time step. At the suggestion of [21] following ratios are used to determine the movement and speed:

$$\begin{aligned}\{\dot{q}\}^{j+1} &= \{\dot{q}\}^j + \tau[(1-\gamma)\{\ddot{q}\}^j + \gamma\{\ddot{q}\}^{j+1}]; \\ \{q\}^{j+1} &= \{q\}^j + \tau\{\dot{q}\}^j + \tau^2[(\frac{1}{2}-\beta)\{\ddot{q}\}^j + \beta\{\ddot{q}\}^{j+1}];\end{aligned}\quad (6)$$

where  $\gamma$  characterizes the damping schematics  $\gamma=1/2$  in which the damping is taken into account. Sootnoshenie (5) can be represented in the form of an algebraic system

$$\begin{aligned}[A]\{q\}^{j+1} &= \{R\}^j \quad \text{где} \\ \{R\}^j &= \{F\}^j + (\frac{2}{(\Delta t)^2}[M] - [K])\{q\}^j - \frac{1}{(\Delta t)^2}\{q\}^{j-1};\end{aligned}\quad (7)$$

implement variable types vector calculation procedure  $\{q(t)\}$ .

Then, in cases of diagonal matrix elements of the mass matrix of the system it is also diagonal. for the integration time step is assumed to be  $0,125 \cdot 10^{-4}$  with a minimum period of free oscillation element  $6,28 \cdot 10^{-4}$  c.

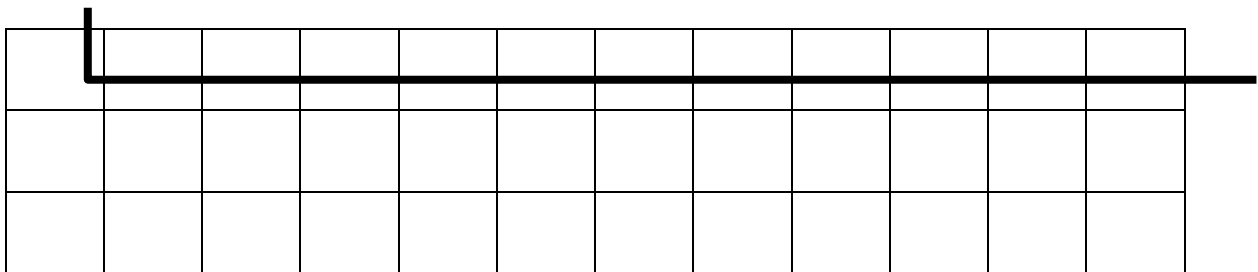
### The solution of the problem of the impact of a plane longitudinal elastic waves in the round cavity

The flat area is considered a rectangular Cartesian coordinate system, which is set to a round hole (Figure 1). The initial conditions are taken zero, which corresponds to the absence of elastic displacements and speeds at  $t=0$ . At  $0 \leq n \leq 10$  ( $n = t/\Delta t$ ) and elastic displacement speed varies from 0 to  $p = \sigma_o / (pc_p)$ ,  $\sigma_o = -0,1$  (MPa), at  $n=10$   $u=p$ , which corresponds to the impact of a plane longitudinal elastic wave such as the Heaviside function  $\sigma_{01}$  (Fig.3)

$$\sigma_{xx}^0 = \sigma_0 H(t), \quad \sigma_{yy}^0 = \sigma_0 \frac{\nu}{1-\nu} H(t), \quad (8)$$

where  $t = (x+R)/c_p$ , ( $\sigma_0 = 1 \text{ MPa}$ );  $c_p$ -longitudinal wave velocity;  $R$ -hole radius. The calculations were performed under the following initial data:  $H=2,0$  m;

$\Delta t = 0,407 \cdot 10^{-5}$  c;  $E = 0,36 \cdot 10$  Mpa;  $\nu = 0,36$ ;  $\rho = 0,122 \cdot 10^4$  kg/m<sup>3</sup>;  $c = 1841$  m/c;  $n = t/\Delta t$ .



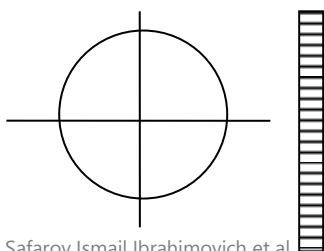
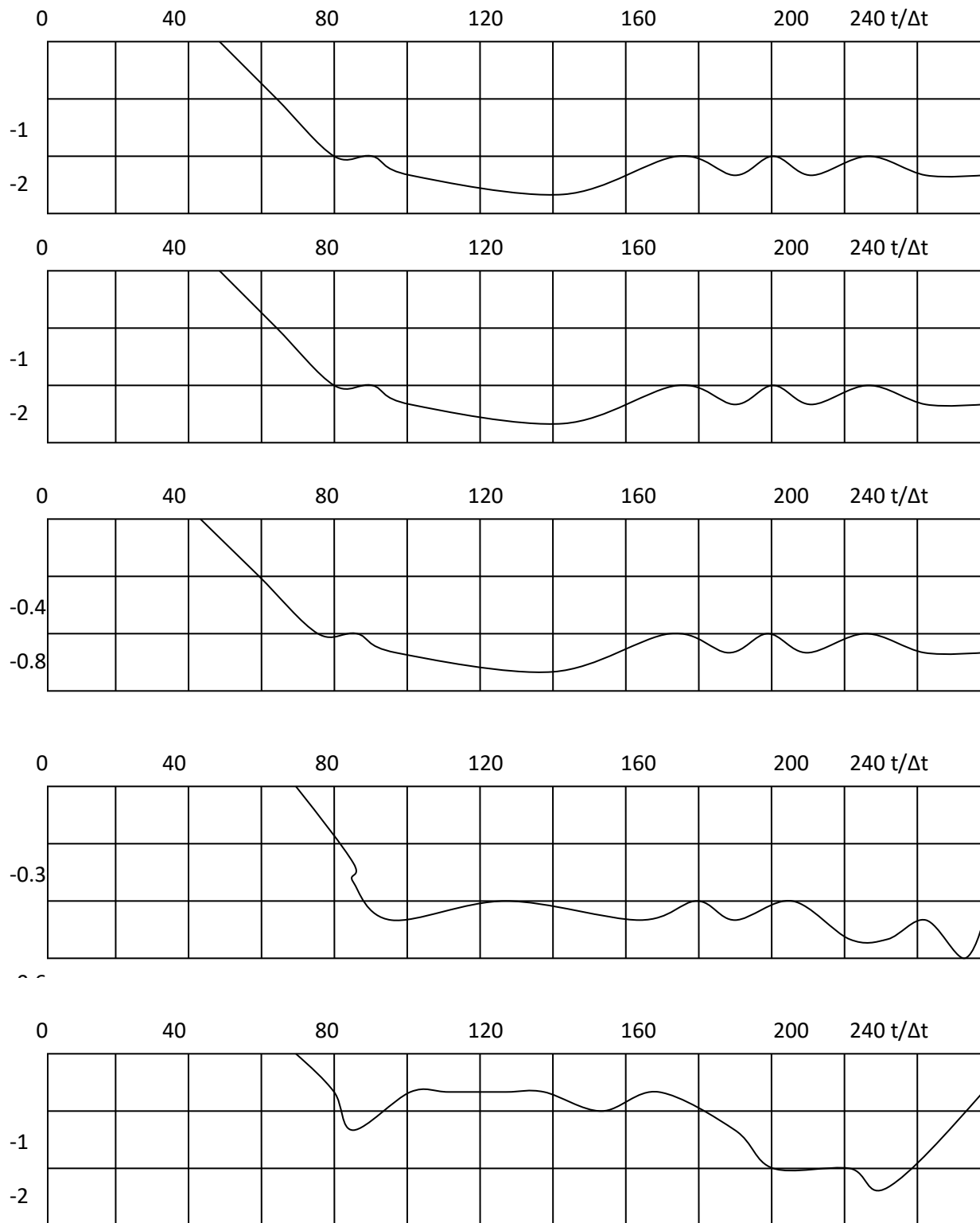
**Figure 3** The impact of the type of the Heaviside function

The study estimated the area has 1536 grid points. Contour circular hole approximated 28 nodal points. The diameter of the circular hole flat front impact extends beyond  $n = 24$ . Charts show that the numerical solution accurately reproduces the wave picture. The discrepancy for the maximum compressive stress of the elastic loop  $\sigma_k$  is 6%. Fig. 4 shows the change in the elastic loop voltage  $\sigma_k$  at points 1A - 5A in  $t_{\text{time}}/\Delta t$ , when exposed to (8) compressing the elastic strain contour  $\sigma_k$  at 1A rises to a maximum and then

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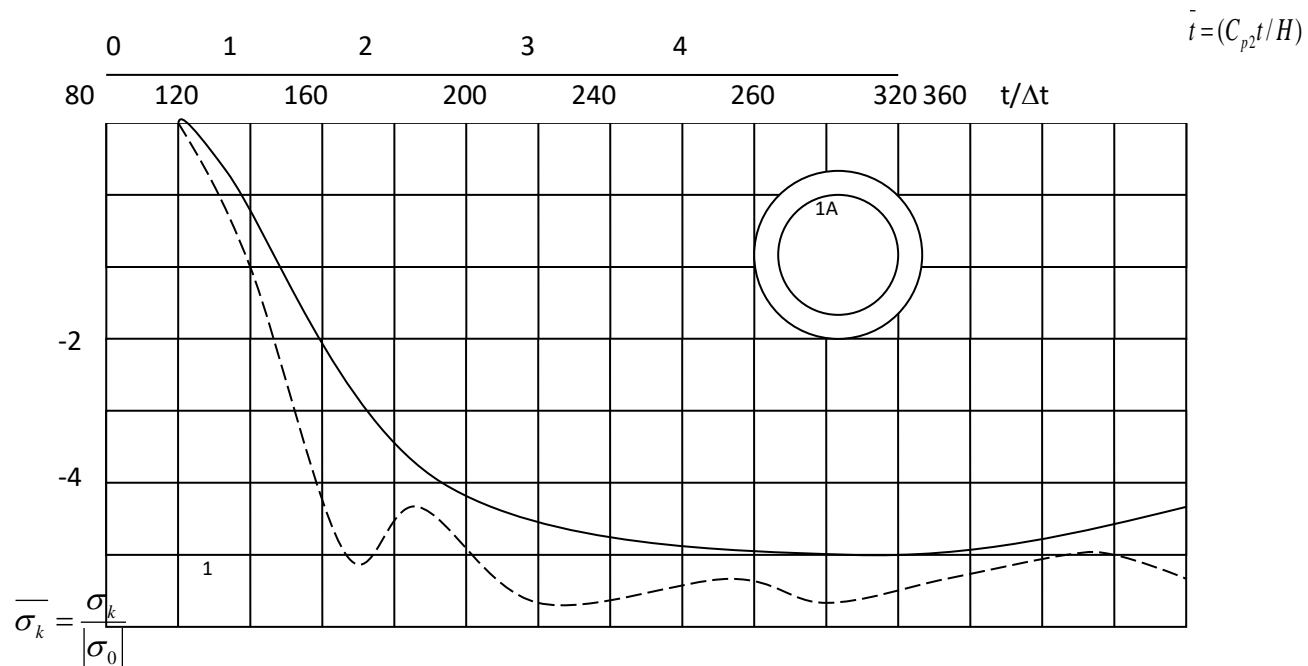
Numerical solution of the problem on the impact plane non-stationary elastic waves by a cylindrical body,

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**Figure 4.** Changing the compression elastic loop voltage  $\bar{\sigma}_x$  at the points in time  $5\Delta t/\Delta t$  on the path of free circular hole under the influence of a plane longitudinal elastic wave type Functions Heaviside.

oscillates asymptotically approaches a constant value. Multiple superposition of direct, reflected and diffracted elastic waves results in a concentration of compressive stress  $\sigma_k$  elastic loop in the vicinity of 1A. The maximum value of the compressive elastic strain contour  $\sigma_k 1A$  reaches the point in nearly three-wave pass edge circular hole diameter and the same  $\sigma_k = -2,712$ . The graphs show that the elastic stress state around a circular hole is committed to the appropriate nominal elastic stress.



**Figure 5**

Changing the compression elastic loop voltage  $\bar{\sigma}_x 1A$  at the points in time  $\bar{t}$  on the path of free round holes: 1 Results analicheskogo solutions under the influence of a plane longitudinal elastic wave type Heaviside certain functions; numerical solution of 2 results in FEM movements under the influence of a plane longitudinal elastic wave type Functions Heaviside.

### *The impact of elastic waves on supported by a round hole*

The Cartesian coordinate system is considered flat area, which is set free round hole. We consider the reinforcement of the middle loop diameter to thickness ratio equal to ten. The initial conditions are taken zero, which corresponds to the absence of elastic displacement and elastic displacement velocities at  $t = 0$ . In the section on distance 2.He (Figure 1) at  $0 \leq n \leq 10$  ( $n = t/\Delta t$ ) elastic displacement and speed to vary linearly from 0 to  $p = \sigma_0 / (\rho_2 c_{p2})$ ,  $\sigma_0 = -0,1 \text{ Mpa}$  (-1kgs/sm),  $atn > 10$   $u = p$ , which corresponds to the impact of a plane longitudinal elastic wave such as the Heaviside function  $\sigma_0$ . The calculations were performed under the following initial data:  $H = 2,0 \text{ m}$ .

$$\Delta t_1 = 0,186 \cdot 10^{-5} \text{ s}; E_1 = 0,72 \cdot 10^5 \text{ Mpa} (0,72 \cdot 10^6 \text{ kg/sm}); \nu = 0,3; p_1 = 0,275 \cdot 10^4 \text{ kg / m}$$

$$(0,6275 \cdot 10^{-5} \text{ kg/s /sm}); c_{p1} = 536 \text{ m/s}; \Delta t_2 = 0,407 \cdot 10^{-5} \text{ s}; E_2 = 0,36 \cdot 10^5 \text{ Mpa}$$

$$(0,36 \cdot 10^5) \cdot 5 \quad \nu = 0,36; p_2 = 0,122 \cdot 10^4 \text{ kg/m}^3 (0,122 \cdot 10^{-5} \text{ kgs/sm}^4); c_{p2} = 1841 \text{ m/s};$$

(...1 – reinforcements, ...2 – environment). The study estimated the area has 1536 nodes to cheh. Inner loop supported by a circular aperture approximated 28 nodal points. According to the thickness of the reinforcement approximated by two nodes. The outer contour supported by a circular aperture approximated 32 nodal points. The diameter of the middle path supported by a circular

aperture plane front impact prohodt for  $n = 60$ . Figure 5 shows the variation of the compressive stress of the elastic loop  $\sigma_{\kappa} (\sigma_{\kappa} = \sigma_{\kappa} / |\sigma_o|)$  to chke1A in in time  $\bar{t} (t = (c_{p2}t) / H\%)$ . 1(–)–2(-- --) – results of the analytical and numerical solutions under the influence of (8). The discrepancy for the maximum compressive stress of the elastic loop  $\bar{\sigma}_x$  amounts to 16%.

### The impact of a plane elastic wave to reinforce the square hole

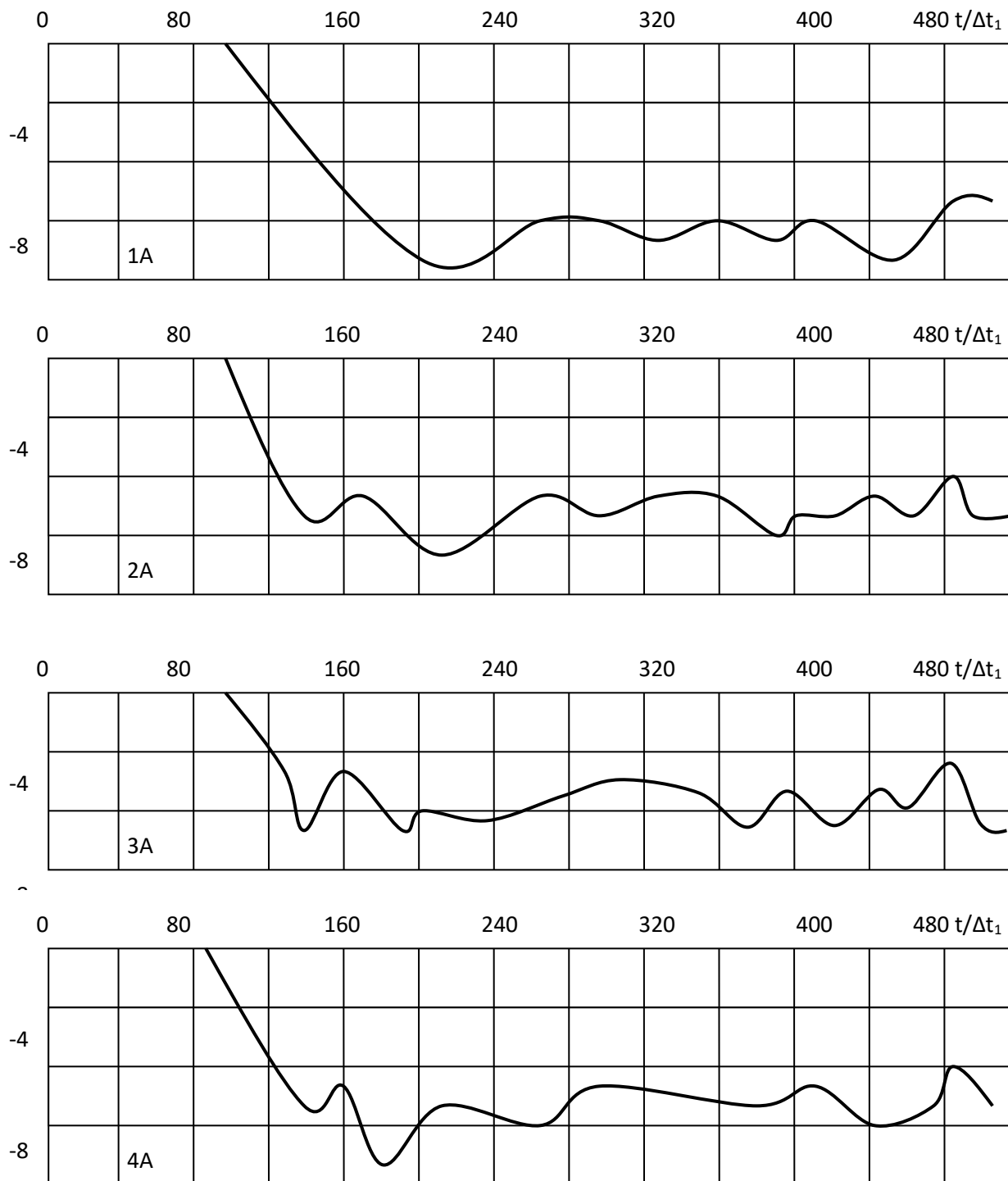
Consider the problem of interaction of plane longitudinal elastic wave to reinforce the square hole. The Cartesian coordinate system is considered flat area, which is set supported by a square hole. We consider the reinforcements with the attitude of the parties to the average contour equal to ten thickness. The initial conditions are taken zero, which corresponds to the absence of elastic displacement and elastic displacement velocities at  $t = 0$ . In section 2. It distance at  $0 \leq n \leq 10$  ( $n = t/\Delta t$ ) elastic displacement and velocity varies linearly from 0 to  $P = \sigma_0 \cdot (\rho_2 c_{p2})$  ( $\sigma_0 = -0,1$  Mpa (-1kgs.sm)),  $atn > 10$   $u_2 = p$ , which corresponds to the impact of a plane longitudinal elastic wave such as the Heaviside function (Figure 3). The calculations were performed under the following initial data:  $H = 2,0$ ;  $\Delta t = 0,186 \cdot 10^{-5}$  c;  $E_1 = 0,72 \cdot 10^5$  Mpa ( $0,72 \cdot 10^6$  kgs/sm<sup>2</sup>);  $\nu = 0,3$ .

The study estimated the area has 13-40 nodal points. Inner loop supported by 36 square hole approximated nodal points. The outer contour supported by a square hole approximated 44 nodal points. The diameter of the middle path supported by a circular aperture plane front impact extends beyond  $n = 60$ . In Figure 6-7 shows the change in the elastic loop voltage  $\sigma_{\kappa} (\sigma_{\kappa} = \sigma_{\kappa} / |\sigma_o|)$  the points 11A-1A in  $t / \Delta t$  pri exposure time (8); The compressive elastic strain contour  $\bar{\sigma}_x$  at 5A rises to a maximum and then oscillates asymptotically approaches a constant value. Permanent superposition of direct, reflected and diffracted elastic waves results in a concentration of compressive elastic loop voltage  $\sigma_{\kappa}$  5A in the vicinity of the point. The maximum value of the compressive stress elastic contour  $\sigma_{\kappa}$  it reaches the point in nearly two passes frontnom-wave part of the middle path and supported by square vent of the same  $\sigma_{\kappa} = -13,9$ . Charts show that the compressive elastic normal stress reinforcements compresses the elastic stress concentration around the holes. The elastic stress state backed by the distance from the square hole is committed to the appropriate nominal elastic stresses. Comparative analysis contour stresses unsupported and supported by the hole (point 1A).

## 4. CONCLUSIONS

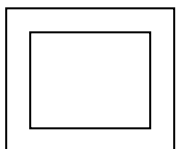
1. under the influence of a plane longitudinal wave type function Heaviside maximum tensile elastic contour stress arises for the free holes at a point on the axis of symmetry in a shady area of internal reinforcement loop; To reinforce the square hole at a point on the axis of symmetry in the illuminated region of the internal circuit reinforcements.
2. with the impact of a plane longitudinal elastic wave such as the Heaviside function to free a round hole and square hole on the free contour compressive elastic strain reaches a maximum value of not more than three passes of the characteristic size of the wave front.
3. Analysis of the numerical results show that the IEC in movements can be successfully applied for the solution of the plane dynamic problem of elasticity and becomes competitive with other methods of dynamic elasticity theory. Studies convergence and stability, comparing the results of numerical solution of the plane dynamic problem of elasticity obtained FEM movements, with the results of analytical solutions are in good coincidence that led us to the physical reliability of the results of the numerical solution of the plane dynamic problem of elasticity obtained FEM movements.
4. The maximum value of the stress intensity in a rectangular building, interacting with the environment, is achieved at the border, near a corner point and in the interior of the body.
5. The maximum stresses in the reinforced contour holes four times greater than the maximum contour of stress in the free holes. Availability reinforcement reduces stress concentration in the medium.
6. Found that in the beginning supported by the hole and the moment when the wave passes near its radius, is almost evenly uniform compression, then there comes a qualitatively new phase of the movement, which outline the voltage fed and appeared appreciable bending stresses rapidly grow to form a ring opening five waves.

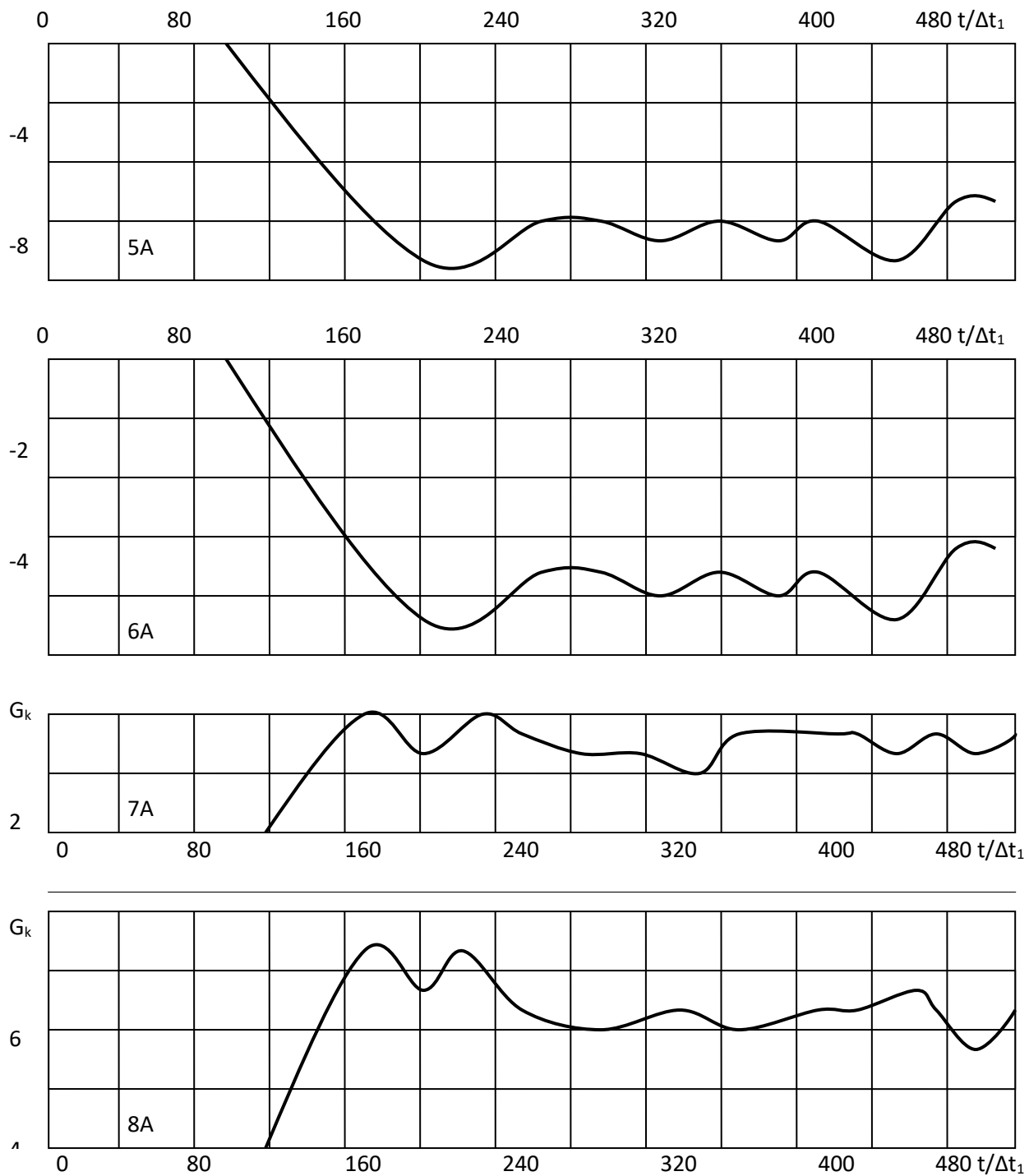




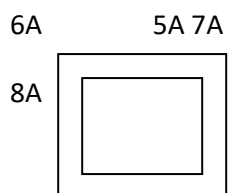
**Figure 6.** Change the compression loop  $\bar{\sigma}_k 1A$  at the points in time  $4At/\Delta t_1$  in the domestic circuit supported by a square hole under the influence of a plane longitudinal elastic wave type Hevsayda function.

1A 2A 3A 4A





**Figure 7.** Changing the loop voltage  $\sigma_k$  5A-8A in time point  $t/\Delta t_1$  in the domestic circuit supported by a square hole under the influence of a plane longitudinal elastic wave type Hevsayda function.



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